DEFINITIONS

Coordinate geometry

Lines, circles and curves described by equations, in Cartesian, parametric and polar form.

26 definitions **5** subtopics By Lucas Potrykus

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HOW TO USE THIS

Each definition gives the exact meaning of a word or symbol. Where there is a symbol, it is shown next to the name, with how to say it out loud. Most definitions have a short example.

Learn definitions word for word: in proofs and in exams, the precise wording is what earns the marks.

Searchable online, with links to every topic, at lemmamathematics.com/definitions.

STRAIGHT LINES

More in **Straight lines and circles**

Cartesian coordinates

A point's position given as (x, y): its distances along the x-axis and y-axis from the origin.

Equation of a straight line

$y = mx + c$ (gradient m, y-intercept c), $y - y_1 = m(x - x_1)$ (through (x_1, y_1)), or $ax + by + c = 0$.

Midpoint

The point halfway between (x_1, y_1) and (x_2, y_2): $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

Distance between two points

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, by Pythagoras.

Parallel lines

Lines with equal gradients.

Perpendicular lines

Lines at right angles. Their gradients multiply to -1: $m_1 m_2 = -1$.

> EXAMPLE
> Perpendicular to gradient $\frac{2}{3}$ is gradient $-\frac{3}{2}$.

Perpendicular bisector

The line through the midpoint of a segment at right angles to it. Every point on it is the same distance from both ends.

Collinear

Three or more points are collinear if they lie on one straight line.

CIRCLES

More in **Straight lines and circles**

Equation of a circle

$(x-a)^2+(y-b)^2=r^2$ is the circle with centre (a,b) and radius r. The expanded form $x^2+y^2+2fx+2gy+c=0$ has centre $(-f,-g)$.

Chord

A straight line segment joining two points on a curve. The perpendicular bisector of a chord of a circle passes through the centre.

Tangent to a circle

A line touching the circle at exactly one point. It is perpendicular to the radius at that point.

Diameter

A chord through the centre. The angle in a semicircle is a right angle (Thales).

Circumcircle

The circle through all three vertices of a triangle. Its centre is where the perpendicular bisectors of the sides meet.

PARAMETRIC EQUATIONS

More in **Parametric equations**

Parameter

A third variable (often t or θ) used to describe both coordinates of a curve: $x=f(t)$, $y=g(t)$.

Parametric equations

A pair $x=f(t)$, $y=g(t)$ giving a point of the curve for each value of the parameter.

> EXAMPLE
> $x=3\cos t$, $y=3\sin t$ is the circle of radius 3.

Cartesian equation

An equation connecting x and y directly, with no parameter. Found by eliminating the parameter.

> EXAMPLE
> $x=t^2$, $y=2t$ gives $y^2=4x$.

Parametric differentiation

$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{\mathrm{d}y/\mathrm{d}t}{\mathrm{d}x/\mathrm{d}t}$, valid where $\frac{\mathrm{d}x}{\mathrm{d}t}\neq 0$.

POLAR COORDINATES

More in **Polar coordinates**

Polar coordinates

A point's position given as (r, θ): its distance r from the pole and the angle θ anticlockwise from the initial line. $x = r\cos\theta$, $y = r\sin\theta$ and $r^2 = x^2 + y^2$.

Pole and initial line

The pole is the origin of polar coordinates; the initial line is the half-line from it where $\theta = 0$, usually the positive x-axis.

Area in polar coordinates

The area swept out by a polar curve from $\theta = \alpha$ to $\theta = \beta$ is $\frac{1}{2}\int_{\alpha}^{\beta} r^2 \,\mathrm{d}\theta$.

Cardioid

The heart-shaped curve $r = a(1 + \cos\theta)$.

CONIC SECTIONS

More in **Roots of polynomials and series**

Conic section

A curve formed where a plane cuts a double cone: a circle, ellipse, parabola or hyperbola.

Parabola

The set of points equidistant from a fixed point (the **focus**) and a fixed line (the **directrix**). Standard form $y^2 = 4ax$, with focus $(a, 0)$ and directrix $x = -a$.

Ellipse

A stretched circle, $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$. The sum of the distances from any point on it to the two foci is constant.

Hyperbola

The curve $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$, with two branches and asymptotes $y = \pm\frac{b}{a}x$. The rectangular hyperbola $xy = c^2$ has perpendicular asymptotes.

Eccentricity

A number $e \geq 0$ measuring how far a conic is from a circle: $e = 0$ circle, $0 < e < 1$ ellipse, $e = 1$ parabola, $e > 1$ hyperbola.

Spotted a mistake, or a definition that should be here? Email lpotry31@gmail.com. More at lemmamathematics.com.