DEFINITIONS

Numerical methods

Finding roots and areas approximately, and measuring how accurate an answer is.

17 definitions **3** subtopics By Lucas Potrykus

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HOW TO USE THIS

Each definition gives the exact meaning of a word or symbol. Where there is a symbol, it is shown next to the name, with how to say it out loud. Most definitions have a short example.

Learn definitions word for word: in proofs and in exams, the precise wording is what earns the marks.

Searchable online, with links to every topic, at lemmamathematics.com/definitions.

LOCATING AND FINDING ROOTS

More in **Numerical methods**

Change of sign

If f is continuous on $[a, b]$ and $f(a)$, $f(b)$ have opposite signs, there is a root between a and b. It can fail for a discontinuous function, and misses roots where the sign does not change.

Continuous function

A function whose graph has no breaks, jumps or holes over an interval: it can be drawn without lifting the pen.

Iteration

Repeating a process, feeding each output back in as the next input: $x_{n+1} = g(x_n)$.

Fixed-point iteration

Rearranging $f(x) = 0$ as $x = g(x)$ and iterating $x_{n+1} = g(x_n)$. It converges to a root α if $|g'(\alpha)| < 1$ (and you start close enough).

Staircase and cobweb diagrams

Pictures of fixed-point iteration on the graphs of $y = x$ and $y = g(x)$. A staircase approaches from one side ($0 < g' < 1$); a cobweb spirals in ($-1 < g' < 0$).

Newton–Raphson method

$x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$: follow the tangent at x_n down to the axis. Usually very fast, but it fails if $f'(x_n)$ is near 0.

Interval bisection

Repeatedly halving an interval that contains a sign change, keeping the half where the sign still changes.

More in **Further numerical methods**

Linear interpolation (false position)

Approximating a root by where the chord between $(a, f(a))$ and $(b, f(b))$ crosses the x-axis.

More in **Further numerical methods**

Order of convergence

How fast an iteration's error shrinks. Linear convergence multiplies the error by roughly a constant each step; Newton–Raphson is quadratic, roughly squaring it.

More in **Further numerical methods**

NUMERICAL INTEGRATION AND DIFFERENTIAL EQUATIONS

More in **Further numerical methods**

Trapezium rule

$\int_a^b y\,\mathrm{d}x \approx \frac{h}{2}\big(y_0 + 2(y_1 + \cdots + y_{n-1}) + y_n\big)$ with strips of width $h = \frac{b-a}{n}$. It overestimates for a convex curve and underestimates for a concave one.

More in **Numerical methods**

Ordinate

The y-value (height) of a curve at a given x. "$n+1$ ordinates" means n strips.

More in **Numerical methods**

Mid-ordinate rule

$\int_a^b y\,\mathrm{d}x \approx h\left(y_{\frac{1}{2}} + y_{\frac{3}{2}} + \cdots + y_{n-\frac{1}{2}}\right)$, using the heights at the middle of each strip.

Simpson's rule

$\int_a^b y\,\mathrm{d}x \approx \frac{h}{3}\big(y_0 + 4(y_1 + y_3 + \cdots) + 2(y_2 + y_4 + \cdots) + y_n\big)$ with an even number of strips. It fits parabolas instead of straight lines.

Euler's method

A step-by-step solution of $\frac{\mathrm{d}y}{\mathrm{d}x} = f(x, y)$: $y_{n+1} = y_n + hf(x_n, y_n)$, following the tangent for a step of size h.

ACCURACY AND ERROR

More in **Numerical methods**

Decimal places and significant figures

Rounding to n decimal places keeps n digits after the point; to n significant figures keeps n digits counted from the first non-zero one.

> EXAMPLE
>
> 0.004718 is 0.0047 to 2 s.f. and 0.005 to 3 d.p.

Upper and lower bounds

The largest and smallest values a rounded number could have been. 7.4 to 1 d.p. lies in $[7.35, 7.45)$.

Absolute, relative and percentage error

Absolute error = approximation − exact value. Relative error = absolute error ÷ exact value; percentage error is that times 100.

Spotted a mistake, or a definition that should be here? Email lpotry31@gmail.com. More at lemmamathematics.com.