DEFINITIONS

Probability

Events, how to combine them, and how to update probabilities with new information.

19 definitions **2** subtopics By Lucas Potrykus

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HOW TO USE THIS

Each definition gives the exact meaning of a word or symbol. Where there is a symbol, it is shown next to the name, with how to say it out loud. Most definitions have a short example.

Learn definitions word for word: in proofs and in exams, the precise wording is what earns the marks.

Searchable online, with links to every topic, at lemmamathematics.com/definitions.

EXPERIMENTS AND EVENTS

More in **Probability**

Experiment and trial

An experiment is a repeatable process with uncertain results; each repetition is a trial.

Outcome

A single possible result of an experiment.

Sample space

The set of all possible outcomes, often written S or Ω.

EXAMPLE

Rolling a dice: $S = \{1, 2, 3, 4, 5, 6\}$.

Event

A set of outcomes, that is a subset of the sample space.

EXAMPLE

"An even score" is the event $\{2, 4, 6\}$.

Probability of an event

$\mathrm{P}(A)$

SAY *P of A; the probability of A*

A number from 0 (impossible) to 1 (certain). $\mathrm{P}(A \cap B)$ is the probability of both; $\mathrm{P}(A \cup B)$ of at least one.

EXAMPLE

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$.

Complementary event

A'

SAY *A prime; not A*

The event A' that A does not happen. $P(A') = 1 - P(A)$.

Equally likely outcomes

When all n outcomes are equally likely, $P(A) = \frac{\text{number of outcomes in } A}{n}$.

Relative frequency

The proportion of trials in which an event happened, used as an estimate of its probability. It settles down as the number of trials grows.

Expected frequency

The number of times you would expect an event in n trials: $n \times P(A)$.

COMBINING EVENTS

More in **Probability**

Mutually exclusive

Events that cannot happen together: $P(A \cap B) = 0$, so $P(A \cup B) = P(A) + P(B)$.

Exhaustive

Events that between them cover the whole sample space, so at least one must happen.

Independent events

Events where knowing that one happened does not change the probability of the other: $P(A \cap B) = P(A)P(B)$, equivalently $P(A \mid B) = P(A)$.

Addition rule

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ for any two events.

Conditional probability

$\mathrm{P}(A \mid B)$

SAY *P of A given B*

The probability of A once you know B has happened: $\mathrm{P}(A \mid B) = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$. In general $\mathrm{P}(A \mid B) \neq \mathrm{P}(B \mid A)$. If $\mathrm{P}(A \mid B) = \mathrm{P}(A)$, the events are independent.

> EXAMPLE
> Roll a fair die: $\mathrm{P}(\mathbf{6} \mid \mathbf{even}) = \frac{1}{3}$.

Conditional probability

$P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$: the probability of A once you know B has happened.

Multiplication rule

$P(A \cap B) = P(A)P(B \mid A)$: multiply along the branches of a tree diagram.

Tree diagram

A branching diagram for events in stages, with probabilities on the branches. Multiply along a path, add between paths.

Two-way table

A table of frequencies (or probabilities) classified by two features at once.

Bayes' theorem

$P(A \mid B) = \dfrac{P(B \mid A)P(A)}{P(B)}$, which reverses a conditional probability.

Spotted a mistake, or a definition that should be here? Email lpotry31@gmail.com. More at lemmamathematics.com.