DEFINITIONS

Sets and number systems

Collections of objects, the standard sets of numbers, and intervals of the number line.

33 definitions **4** subtopics By Lucas Potrykus

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HOW TO USE THIS

Each definition gives the exact meaning of a word or symbol. Where there is a symbol, it is shown next to the name, with how to say it out loud. Most definitions have a short example.

Learn definitions word for word: in proofs and in exams, the precise wording is what earns the marks.

Searchable online, with links to every topic, at lemmamathematics.com/definitions.

SETS

Set

A collection of distinct objects, called its elements or members. A set is determined entirely by which elements it contains.

Set { }

SAY *the set containing …*

Curly brackets list the elements of a set. Order and repetition do not matter, so $\{1, 2\} = \{2, 1\} = \{1, 1, 2\}$.

EXAMPLE

$\{2, 4, 6, 8\}$ is the set of even numbers from 2 to 8.

Set-builder notation $\{x : P(x)\}$

SAY *the set of all x such that $P(x)$*

Describes a set by a property its elements share. The colon is read "such that"; a vertical bar, $\{x \mid P(x)\}$, means the same.

EXAMPLE

$\{x \in \mathbb{R} : x^2 < 4\}$ is the interval $(-2, 2)$.

Is an element of $\in$

SAY *is in; is an element of; belongs to*

$x \in A$ says that x is one of the objects in the set A.

EXAMPLE

$3 \in \{1, 2, 3\}$ and $\sqrt{2} \in \mathbb{R}$.

Is not an element of

$\notin$

SAY *is not in*

$x \notin A$ says that x is not in the set A.

> EXAMPLE
> $\sqrt{2} \notin \mathbb{Q}$: the square root of 2 is irrational.

More in **Proof I**

Empty set

$\varnothing$

SAY *the empty set*

The set with no elements, also written $\{\}$. Careful: $\{\varnothing\}$ is not empty, because it has one element (the empty set itself).

> EXAMPLE
> $\{x \in \mathbb{R} : x^2 = -1\} = \varnothing$.

Universal set

ξ

SAY *"ksy" or "zai"*

The set of everything under consideration, drawn as the rectangle in a Venn diagram. Also written $\mathcal{E}$ or U.

> EXAMPLE
> With $\xi = \{1, 2, 3, 4, 5, 6\}$ for a dice roll, the event "even" is $\{2, 4, 6\}$.

More in **Probability**

Equal sets

Two sets are equal when they have exactly the same elements. To prove $A = B$, show $A \subseteq B$ and $B \subseteq A$.

Is a subset of

$\subseteq$

SAY *is a subset of; is contained in*

$A \subseteq B$ means every element of A is also in B. Every set is a subset of itself, and $\varnothing$ is a subset of every set.

> EXAMPLE
> $\{1, 2\} \subseteq \{1, 2, 3\}$ and $\mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$.

Is a proper subset of

$\subset$

SAY *is a proper subset of*

$A \subset B$ usually means $A \subseteq B$ and $A \neq B$. Some books use $\subset$ for any subset and write $\subsetneq$ for a proper one, so check the convention.

> EXAMPLE
> $\{1, 2\} \subset \{1, 2, 3\}$, but $\{1, 2, 3\} \not\subset \{1, 2, 3\}$.

Finite and infinite sets

A set is finite if it has a whole number of elements, and infinite otherwise.

> EXAMPLE
> $\{x \in \mathbb{Z} : x^2 < 10\}$ is finite (7 elements); the set of primes is infinite.

Number of elements

$|A|$

SAY *the size of A; the cardinality of A*

How many elements A has. At school this is often written $n(A)$. For infinite sets, comparing sizes leads to the surprising world of cardinals (see $\aleph_0$ below).

> EXAMPLE
> $|\{1, 2, 3\}| = 3$ and $|\varnothing| = 0$.

Power set

$\mathcal{P}(A)$

SAY *the power set of A*

The set of all subsets of A. If $|A| = n$ then $|\mathcal{P}(A)| = 2^n$, since each element is either in a subset or not.

> EXAMPLE
> $\mathcal{P}(\{1, 2\}) = \{\varnothing, \{1\}, \{2\}, \{1, 2\}\}$.

OPERATIONS ON SETS

More in **Probability**

Union

$\cup$

SAY *union; "or"*

$A \cup B$ is the set of elements in A, in B, or in both.

> EXAMPLE
> $\{1, 2, 3\} \cup \{2, 3, 4\} = \{1, 2, 3, 4\}$.

Intersection

$\cap$

SAY *intersection; "and"*

$A \cap B$ is the set of elements in both A and B. If $A \cap B = \varnothing$ the sets are called disjoint (in probability, mutually exclusive).

> EXAMPLE
> $\{1, 2, 3\} \cap \{2, 3, 4\} = \{2, 3\}$.

Complement

A'

SAY *A dash; not A; A complement*

Everything in the universal set that is not in A. Also written A^c or $\overline{A}$.

> EXAMPLE
> With $\xi = \{1, \ldots, 6\}$ and $A = \{1, 2\}$, $A' = \{3, 4, 5, 6\}$. In probability, $\mathrm{P}(A') = 1 - \mathrm{P}(A)$.

Set difference

$A \setminus B$

SAY *A minus B; A without B*

The elements of A that are not in B. Also written $A - B$. It equals $A \cap B'$.

> EXAMPLE
> $\{1, 2, 3\} \setminus \{2, 3, 4\} = \{1\}$, and $\mathbb{R} \setminus \{0\}$ is the set of non-zero reals.

Disjoint sets

Sets with no elements in common: $A \cap B = \emptyset$.

> EXAMPLE
> The even and the odd integers are disjoint.

Venn diagram

A diagram showing sets as overlapping regions inside a rectangle that represents the universal set.

Cartesian product

$A \times B$

SAY *A cross B*

The set of all ordered pairs (a, b) with $a \in A$ and $b \in B$. If A has m elements and B has n, then $A \times B$ has mn.

> EXAMPLE
> $\mathbb{R} \times \mathbb{R} = \mathbb{R}^2$ is the coordinate plane; $\{0, 1\} \times \{x, y\} = \{(0, x), (0, y), (1, x), (1, y)\}$.

NUMBER SYSTEMS AND CONSTANTS

Natural numbers

$\mathbb{N}$

SAY *N; the naturals*

The counting numbers. Most school books take $\mathbb{N} = \{1, 2, 3, \ldots\}$, but many university courses include 0. If it matters, $\mathbb{Z}^+$ and $\mathbb{Z}_{\geq 0}$ are unambiguous.

> EXAMPLE
> $5 \in \mathbb{N}$ but $-5 \notin \mathbb{N}$.

Integers

$\mathbb{Z}$

SAY *Z; the integers*

All whole numbers, positive, negative and zero: $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$. The Z is from the German *Zahlen*, "numbers". $\mathbb{Z}^+$ means the positive integers.

> EXAMPLE
> $-7 \in \mathbb{Z}$ but $\frac{1}{2} \notin \mathbb{Z}$.

Rational numbers

$\mathbb{Q}$

SAY *Q; the rationals*

Numbers that can be written as $\frac{p}{q}$ with $p, q \in \mathbb{Z}$ and $q \neq 0$. Q is for "quotient". Their decimals terminate or repeat.

> EXAMPLE
> $0.75 = \frac{3}{4} \in \mathbb{Q}$ and $0.\dot{3} = \frac{1}{3} \in \mathbb{Q}$, but $\pi \notin \mathbb{Q}$.

More in **Proof I**

Real numbers

$\mathbb{R}$

SAY *R; the reals*

Every point on the number line, rational or irrational. $\mathbb{R}^+$ means the positive reals, and $\mathbb{R}^n$ is the set of lists of n real numbers, such as points in space when $n = 3$.

> EXAMPLE
> $\sqrt{2}, \pi, \mathrm{e}, -4 \in \mathbb{R}$, but $\mathrm{i} \notin \mathbb{R}$.

Complex numbers

$\mathbb{C}$

SAY *C; the complex numbers*

Numbers $a + b\mathrm{i}$ with $a, b \in \mathbb{R}$ and $\mathrm{i}^2 = -1$. Every polynomial equation has all its roots in $\mathbb{C}$.

> EXAMPLE
> $3 - 2\mathrm{i} \in \mathbb{C}$, and $\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R} \subseteq \mathbb{C}$.

More in **Complex numbers**

Positive, negative and non-negative

Positive means > 0 and negative means < 0. Non-negative means ≥ 0, so zero is non-negative but not positive.

Number line

A line on which every real number has its own point, increasing from left to right. The real numbers fill it with no gaps.

Pi

π

SAY *pie*

The ratio of a circle's circumference to its diameter, $\pi \approx 3.14159$. It is irrational, and even transcendental (it is not the root of any polynomial with integer coefficients).

> EXAMPLE
> A full turn is 2π radians.

More in **Trigonometry**

Infinity

∞

SAY *infinity*

Not a real number, but a shorthand for "grows without bound". So expressions like $\infty - \infty$ have no fixed value, and you cannot cancel infinities.

> EXAMPLE
> $\sum_{r=1}^{\infty} \frac{1}{2^r} = 1$ means the partial sums get as close to 1 as you like.

More in **Sequences and series**

INTERVALS AND BOUNDS

More in **Functions and graphs**

Closed interval

$[a, b]$

SAY *the closed interval from a to b*

All real x with $a \leq x \leq b$: square brackets include the endpoint.

> EXAMPLE
> $[0, 1]$ contains 0, 1 and everything in between.

Open interval

(a, b)

SAY *the open interval from a to b*

All real x with $a < x < b$: round brackets exclude the endpoint. Also written $]a, b[$ in some countries. Infinity always gets a round bracket, as in $[0, \infty)$. Context tells you whether (a, b) is an interval or a point.

> EXAMPLE
> $x \in (2, 5]$ means $2 < x \leq 5$.

Half-open interval

An interval containing one endpoint but not the other: $[a, b) = \{x : a \leq x < b\}$ and $(a, b] = \{x : a < x \leq b\}$.

Bounded

A set of numbers is bounded above if some number M is at least as large as every element (an upper bound), bounded below if some number is no larger than every element, and bounded if both.

> EXAMPLE
> $(0, 1)$ is bounded; $\mathbb{N}$ is bounded below but not above.

Spotted a mistake, or a definition that should be here? Email lpotry31@gmail.com. More at lemmamathematics.com.