DEFINITIONS

Vectors

Quantities with size and direction, and the lines and planes they describe.

28 definitions **3** subtopics By Lucas Potrykus

CONTENTS

1. Vectors and scalars 13
2. Products of vectors 6
3. Lines and planes 9

HOW TO USE THIS

Each definition gives the exact meaning of a word or symbol. Where there is a symbol, it is shown next to the name, with how to say it out loud. Most definitions have a short example.

Learn definitions word for word: in proofs and in exams, the precise wording is what earns the marks.

Searchable online, with links to every topic, at lemmamathematics.com/definitions.

VECTORS AND SCALARS

More in **Vectors**

Vector

A quantity with both magnitude (size) and direction, such as displacement, velocity or force.

Scalar

A quantity with magnitude only, such as distance, speed, mass or time. Multiplying a vector by a scalar changes its length (and reverses it if the scalar is negative).

A vector

$\mathbf{a}$, $\underline{a}$, $\vec{a}$

SAY *vector a*

Three ways to show that $\boldsymbol{a}$ is a vector, not a number. Printed books use **bold**; by hand, underline it (bold is hard to write).

The vector from A to B

$\overrightarrow{AB}$

SAY *vector A B*

The displacement from point A to point B. If A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$, then $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$ ("end minus start").

> EXAMPLE
> $A = (1, 2)$ and $B = (4, 6)$ give $\overrightarrow{AB} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$.

Column vector

$\begin{pmatrix} x \\ y \\ z \end{pmatrix}$

SAY *the column vector x, y, z*

Lists a vector's components. The same vector is $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$.

EXAMPLE

$\begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$.

Standard unit vectors

$\mathbf{i}, \mathbf{j}, \mathbf{k}$

SAY *i, j, k*

Unit vectors in the directions of the x-, y- and z-axes. Not to be confused with the complex number $\mathbf{i}$.

Magnitude of a vector

$|\mathbf{a}|$

SAY *the magnitude of a; mod a*

The length of the vector, found by Pythagoras. At university it is often written $\|\mathbf{a}\|$ and called the norm.

EXAMPLE

$\left|\begin{pmatrix} 3 \\ 4 \end{pmatrix}\right| = 5$.

Unit vector

$\hat{\mathbf{a}}$

SAY *a hat*

The vector of length 1 in the direction of $\mathbf{a}$: $\hat{\mathbf{a}} = \frac{\mathbf{a}}{|\mathbf{a}|}$.

EXAMPLE

If $\mathbf{a} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ then $\hat{\mathbf{a}} = \begin{pmatrix} 0.6 \\ 0.8 \end{pmatrix}$.

Zero vector

The vector $\mathbf{0}$ with magnitude 0 and no direction.

Position vector

The vector from the origin to a point: A has position vector $\mathbf{a} = \overrightarrow{OA}$. Then $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$.

Resultant

The single vector equal to the sum of two or more vectors: the overall effect.

Parallel vectors

$\mathbf{a}$ and $\mathbf{b}$ are parallel when $\mathbf{b} = k\mathbf{a}$ for some scalar $k \neq 0$.

Dividing a line in a ratio

The point dividing AB in the ratio $\lambda : \mu$ has position vector $\dfrac{\mu\mathbf{a} + \lambda\mathbf{b}}{\lambda + \mu}$.

PRODUCTS OF VECTORS

More in **Vectors: lines and planes**

Scalar (dot) product

$\mathbf{a} \cdot \mathbf{b}$

SAY *a dot b*

Multiply matching components and add. The answer is a number: $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos\theta$, which is 0 exactly when the vectors are perpendicular.

EXAMPLE

$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix} = 4 + 10 + 18 = 32.$$

More in **Vectors**

Angle between vectors

$\cos\theta = \dfrac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$. In particular $\mathbf{a} \cdot \mathbf{b} = 0$ exactly when non-zero vectors are perpendicular.

Vector (cross) product

$\mathbf{a} \times \mathbf{b}$

SAY *a cross b*

A vector perpendicular to both $\mathbf{a}$ and $\mathbf{b}$, with length $|\mathbf{a}||\mathbf{b}| \sin\theta$ (the area of the parallelogram they span). Only defined in three dimensions, and $\mathbf{b} \times \mathbf{a} = -\mathbf{a} \times \mathbf{b}$.

EXAMPLE

$\mathbf{i} \times \mathbf{j} = \mathbf{k}$.

Scalar triple product

$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$

$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$, the volume of the parallelepiped with edges $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ (up to sign). It is zero exactly when the three vectors are coplanar.

Coplanar

Points (or vectors) that lie in a single plane.

Linearly independent

Vectors are linearly independent if no one of them can be written as a combination of the others. Three independent vectors in 3D are not coplanar.

LINES AND PLANES

More in **Vectors: lines and planes**

Vector equation of a line

$\mathbf{r} = \mathbf{a} + \lambda\mathbf{d}$

SAY *r equals a plus lambda d*

Every point on the line, as λ runs through all real numbers: start at the point $\mathbf{a}$ and move any multiple of the direction $\mathbf{d}$.

EXAMPLE

$$\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}.$$

More in **Vectors**

Direction vector

A vector parallel to a line, the $\mathbf{d}$ in $\mathbf{r} = \mathbf{a} + \lambda\mathbf{d}$.

Cartesian equation of a line in 3D

$\frac{x-a_1}{d_1}=\frac{y-a_2}{d_2}=\frac{z-a_3}{d_3}$ for the line through (a_1,a_2,a_3) with direction $\begin{pmatrix}d_1\\d_2\\d_3\end{pmatrix}$.

Skew lines

Lines in 3D that are not parallel and never meet.

Equation of a plane

$\mathbf{r}\cdot\mathbf{n}=\mathbf{a}\cdot\mathbf{n}$ (normal form), $\mathbf{r}=\mathbf{a}+\lambda\mathbf{b}+\mu\mathbf{c}$ (parametric form), or $ax+by+cz=d$ (Cartesian form, with normal $\begin{pmatrix}a\\b\\c\end{pmatrix}$).

Normal vector

A vector perpendicular to a plane (or to a line in 2D).

Angles with planes

The angle between two planes is the angle between their normals. The angle between a line and a plane is $90°$ minus the angle between the line and the normal.

Shortest distance

The length of the perpendicular. From (x_1,y_1,z_1) to the plane $ax+by+cz=d$ it is $\frac{|ax_1+by_1+cz_1-d|}{\sqrt{a^2+b^2+c^2}}$.

Foot of the perpendicular

The point on a line or plane closest to a given point; the line joining them meets the line or plane at right angles.

Spotted a mistake, or a definition that should be here? Email lpotry31@gmail.com. More at lemmamathematics.com.