Name _____ Date _____ Score _____ / 100

LEMMA PRACTICE PAPER

Level 1

Pure Mathematics

Paper 1 · Foundation

Paper reference **LP1/P** Time **2 hours**

TOTAL MARKS	100

You must have: pen, pencil, ruler, graph paper and a calculator.

INSTRUCTIONS

- Use black ink or ball-point pen; pencil may be used for diagrams.
- Answer **all** questions, in the spaces provided. If you need more space, continue on extra paper and label it clearly.
- Show all your working. An answer with no supporting method may get no marks.
- Give answers to 3 significant figures unless a question says otherwise, and give exact answers when asked.

INFORMATION

- There are **15** questions, and the paper is out of **100** marks.
- The marks for each part of a question are shown in brackets, for example **(3)**. Use them to judge how much to write.
- The questions are in roughly increasing order of difficulty.
- Difficulty: **Foundation**. Routine, single-method questions with clear instructions. If you have covered the topic, you should be able to start every question. Good for building confidence and checking the basics.

ADVICE

- Read each question carefully before you start.
- Keep an eye on the time: roughly one minute per mark.
- Check your answers if you have time at the end.
- The mark scheme, very detailed model solutions and video explanations are free at **lemmamathematics.com/papers/pure-1**.

1 In this question you must show all stages of your working. Solutions relying entirely on calculator technology are not acceptable.

(a) Simplify fully $(2x^3)^2 \times 3x^{-1}$. **(2)**

(b) Write $\sqrt{48} + \sqrt{27}$ in the form $a\sqrt{3}$, where a is an integer. **(2)**

(c) Find the exact value of $8^{-\frac{2}{3}}$. **(2)**

(Total for Question 1 is 6 marks)

2 Solve the simultaneous equations **(7)**

$$y = x + 1, \qquad x^2 + y^2 = 13.$$

(Total for Question 2 is 7 marks)

3 The points A and B have coordinates $(1, 3)$ and $(5, 11)$ respectively.

(a) Find an equation of the line AB, giving your answer in the form $y = mx + c$. **(3)**

(b) Find an equation of the perpendicular bisector of AB, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(3)**

(Total for Question 3 is 6 marks)

4 The points A and B have coordinates $(1, 3)$ and $(5, 11)$. The perpendicular bisector of AB has equation $x + 2y - 17 = 0$, and it crosses the x-axis at the point C.

(a) Find the coordinates of C. **(1)**

(b) Find the area of triangle ABC. **(3)**

(Total for Question 4 is 4 marks)

5

$$f(x) = x^2 - 6x + 5$$

(a) Write $f(x)$ in the form $(x + p)^2 + q$, where p and q are integers. **(2)**

(b) Hence state the coordinates of the turning point of the curve $y = f(x)$. **(1)**

(c) Solve $f(x) = 0$. **(2)**

(d) Sketch the curve $y = f(x)$, showing the coordinates of any points where it meets the axes and of its turning point. **(2)**

(e) Find the set of values of x for which $f(x) < 0$. **(2)**

(Total for Question 5 is 9 marks)

6 The curve C has equation

$$y = 2x^3 - 3x^2 - 12x + 4.$$

(a) Find $\dfrac{dy}{dx}$. **(2)**

(b) Find the coordinates of the stationary points of C. **(4)**

(c) Determine the nature of each stationary point, giving a reason for your answer. **(3)**

(Total for Question 6 is 9 marks)

7

$$f'(x) = 3x^2 + 4x - 5$$

(a) Find $\int (3x^2 + 4x - 5)\, dx$. **(3)**

(b) Hence find the area of the region bounded by the curve $y = 3x^2 + 4x - 5$, the lines $x = 1$ and $x = 2$, and the x-axis. **(3)**

(Total for Question 7 is 6 marks)

8 The curve $y = f(x)$ has gradient function $f'(x) = 3x^2 + 4x - 5$ and passes through the point $(1, 4)$. **(3)**

Find $f(x)$.

(Total for Question 8 is 3 marks)

9 **(a)** Find the first four terms, in ascending powers of x, of the binomial expansion of $(1 + 2x)^6$, giving each term in its simplest form. **(4)**

(b) Hence find the coefficient of x^2 in the expansion of $(3 - x)(1 + 2x)^6$. **(3)**

(c) Use your expansion from (a), with a suitable value of x, to find an estimate for 1.02^6. Give your answer to 5 decimal places. **(3)**

(Total for Question 9 is 10 marks)

10 **(a)** Solve, for $0 \leq x < 360^\circ$, $\sin x = 0.5$. **(2)**

(b) Solve, for $0 \leq x < 360^\circ$, **(5)**

$$2\cos^2 x + 3\sin x = 3.$$

(Total for Question 10 is 7 marks)

11 In triangle ABC, $AB = 7$ cm, $AC = 5$ cm and angle $BAC = 60^\circ$.

(a) Find the length of BC, giving your answer to 3 significant figures. **(2)**

(b) Find the exact area of triangle ABC. **(2)**

(Total for Question 11 is 4 marks)

12 **(a)** Solve $3^x = 20$, giving your answer to 3 significant figures. **(2)**

(b) Solve $\log_2 x + \log_2(x+2) = 3$. **(4)**

(Total for Question 12 is 6 marks)

13 The number of bacteria in a culture is modelled by $P = 200e^{0.03t}$, where t is the time in hours. **(6)**

(i) State the initial number of bacteria. (ii) Find the time taken for the number of bacteria to reach 500, giving your answer to 3 significant figures. (iii) Find the rate at which the number of bacteria is increasing when $t = 10$.

(Total for Question 13 is 6 marks)

14 The circle C has equation

$$x^2 + y^2 - 6x + 4y - 12 = 0.$$

(a) Find the coordinates of the centre of C and the radius of C. **(3)**

(b) Verify that the point $P(6, 2)$ lies on C. **(1)**

(c) Find an equation of the tangent to C at P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(4)**

(Total for Question 14 is 8 marks)

15 The circle C has equation $(x - 3)^2 + (y + 2)^2 = 25$. The line with equation $y = x + k$, where k is a constant, is a tangent to C.

(a) Find the two possible exact values of k. **(7)**

(b) Hence find the set of values of k for which the line $y = x + k$ meets C at two distinct points. **(2)**

(Total for Question 15 is 9 marks)

TOTAL FOR PAPER IS 100 MARKS

END OF PAPER