LEMMA PRACTICE PAPER · MARK SCHEME

Level 2

Pure Mathematics

Paper 2 · Standard

GENERAL MARKING GUIDANCE

- All candidates must receive the same treatment: mark positively, and give credit for what is shown rather than penalising what is missing.
- **M** marks: method marks, for knowing and applying a correct method to the problem, even if there are small slips.
- **A** marks: accuracy marks, which can only be awarded if the relevant M mark has been earned.
- **B** marks: unconditional accuracy marks, independent of method.
- **dM**: a method mark that depends on the previous M mark. **ft**: follow through from an earlier error. *****: the answer is given in the question, so full working must be seen with no errors. **cao**: correct answer only.
- Correct alternative methods earn full credit. The model solutions show several of them.

QUESTION	SCHEME	MARKS	NOTES
1 (a)	Critical values from $(x-5)(x+3)=0$	**M1**	
	$x=5,\ x=-3$	**A1**	
	$x<-3$ or $x>5$	**A1ft**	Outside region of their critical values.
(b)	$x<8$	**B1**	
(c)	Combines their (a) and (b) correctly	**M1**	
	$\{x : x<-3\}\cup\{x : 5<x<8\}$	**A1**	
	Total for Question 1	**(6)**	

QUESTION	SCHEME	MARKS	NOTES
2	$n^3-n=n(n^2-1)=(n-1)n(n+1)$	**M1**	
	States these are three consecutive integers	**A1**	
	Argues one of them is even and one is a multiple of 3	**M1**	
	Concludes: divisible by $2\times3=6$, because 2 and 3 have no common factor	**A1**	Complete, convincing proof.
	Total for Question 2	**(4)**	

QUESTION	SCHEME	MARKS	NOTES
3	Tests a value of n and evaluates 2^n+1	**M1**	
	$n=3$: $2^3+1=9=3\times3$, not prime	**A1**	Must say why it isn't prime.

QUESTION	SCHEME	MARKS	NOTES
	Total for Question 3	**(2)**	

QUESTION	SCHEME	MARKS	NOTES
4 (a)	$2(x^2-1)+3$	**M1**	
	$2x^2+1$	**A1**	
(b)	$gf(x) = (2x+3)^2 - 1$	**M1**	
	$4x^2+12x+8$	**A1**	
	Sets equal and solves $2x^2+12x+7=0$	**M1**	
	$x = \dfrac{-6 \pm \sqrt{22}}{2}$	**A1**	
(c)	$y = 2x+3 \Rightarrow x = \dfrac{y-3}{2}$	**M1**	
	$f^{-1}(x) = \dfrac{x-3}{2}$	**A1**	
	Total for Question 4	**(8)**	

QUESTION	SCHEME	MARKS	NOTES
5 (a)	$a+4d=17$	**M1**	
	$\frac{10}{2}(2a+9d)=185$	**M1**	
	Solves simultaneously	**M1**	
	$a=5,\ d=3$	**A1**	
(b)	$\frac{n}{2}\left(10+3(n-1)\right) > 1000$	**M1**	
	$3n^2+7n-2000>0$	**M1**	Three-term quadratic.
	Solves: $n > 24.68\ldots$	**M1**	
	$n=25$	**A1**	Accept with $S_{24}=948$, $S_{25}=1025$ shown.
	Total for Question 5	**(8)**	

QUESTION	SCHEME	MARKS	NOTES
6 (a)	$\dfrac{80}{1-r} = 320$	**M1**	
	$1-r=\frac{1}{4} \Rightarrow r=\frac{3}{4}$	**A1***	Given answer: must be shown.
(b)	$80\times\left(\frac{3}{4}\right)^3$	**M1**	
	33.75	**A1**	
(c)	$S_\infty - S_n = 320\left(\frac{3}{4}\right)^n$	**M1**	
	$320\left(\frac{3}{4}\right)^n < 1$	**M1**	

QUESTION	SCHEME	MARKS	NOTES
	Uses logs: $n > \dfrac{\ln 320}{\ln \frac{4}{3}}$	**M1**	Take care: $\ln \frac{3}{4} < 0$ reverses the inequality.
	$n = 21$	**A1**	
	Total for Question 6	**(8)**	

QUESTION	SCHEME	MARKS	NOTES
7 (a)	£20 000	**B1**	
(b)	$18\,000e^{-0.75} + 2000$	**M1**	
	£10 503	**A1**	
(c)	$18\,000e^{-0.15t} = 4000$	**M1**	
	$-0.15t = \ln \frac{2}{9}$	**M1**	
	$t = 10.03$	**A1**	
(d)	Differentiates: $ke^{-0.15t}$	**M1**	
	$\dfrac{dV}{dt} = -2700e^{-0.15t}$	**A1**	
	£1275 per year	**A1**	Decreasing at about £1275 per year.
(e)	£2000	**B1**	
	Total for Question 7	**(10)**	

QUESTION	SCHEME	MARKS	NOTES
8 (a)	$1 - \cos 2\theta = 2\sin^2\theta$	**M1**	
	$\sin 2\theta = 2\sin\theta\cos\theta$	**M1**	
	$\dfrac{2\sin^2\theta}{2\sin\theta\cos\theta} = \dfrac{\sin\theta}{\cos\theta} = \tan\theta$	**A1***	
(b)	$6\sin\theta\cos\theta = 2\cos\theta$	**M1**	
	$2\cos\theta(3\sin\theta - 1) = 0$	**M1**	Factorises (no division by $\cos\theta$).
	$\theta = \dfrac{\pi}{2}$	**A1**	
	$\sin\theta = \frac{1}{3} \Rightarrow \theta = 0.340$	**M1**	
	$\theta = 0.340,\ 2.802$	**A1**	Both, and $\frac{\pi}{2}$, no extras.
	Total for Question 8	**(8)**	

QUESTION	SCHEME	MARKS	NOTES
9	$(x+h)^2 = x^2 + 2xh + h^2$	**B1**	
	$\dfrac{(x+h)^2 - x^2}{h}$	**M1**	
	$= 2x + h$	**A1**	

QUESTION	SCHEME	MARKS	NOTES
	As $h \to 0$, gradient $\to 2x$, with limiting notation	**A1***	
	Total for Question 9	**(4)**	

QUESTION	SCHEME	MARKS	NOTES
10	$\dfrac{dy}{dx} = 3x^2 - 4$	**M1**	
	Gradient at P is -1	**A1**	
	Tangent $y = -x - 2$	**M1**	
	$x^3 - 4x = -x - 2 \Rightarrow x^3 - 3x + 2 = 0$	**M1**	
	Uses factor $(x-1)^2$: $(x-1)^2(x+2) = 0$	**M1**	
	$(-2, 0)$	**A1**	
	Total for Question 10	**(6)**	

QUESTION	SCHEME	MARKS	NOTES
11 (a)	$u = x,\ \dfrac{dv}{dx} = e^{2x},\ v = \frac{1}{2}e^{2x}$ and the parts structure	**M1**	
	$\frac{1}{2}xe^{2x} - \int \frac{1}{2}e^{2x}\,dx$	**A1**	
	Integrates the second term	**M1**	
	$\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + c$	**A1**	
(b)	Substitutes limits 1 and 0	**M1**	
	$\left(\frac{1}{2}e^2 - \frac{1}{4}e^2\right) - \left(0 - \frac{1}{4}\right)$	**M1**	
	$\dfrac{e^2 + 1}{4}$	**A1**	
	Total for Question 11	**(7)**	

QUESTION	SCHEME	MARKS	NOTES
12	$k(x^2+1)^3$	**M1**	
	$k = 1$ found by differentiating or substitution	**M1**	
	$(x^2+1)^3 + c$	**A1**	
	Total for Question 12	**(3)**	

QUESTION	SCHEME	MARKS	NOTES
13 (a)	$\dfrac{A}{1+x} + \dfrac{B}{1-2x}$ and $7x + 1 = A(1-2x) + B(1+x)$	**M1**	
	Substitutes values or compares coefficients	**M1**	
	$A = -2,\ B = 3$	**A1**	
(b)	$(1+x)^{-1} = 1 - x + x^2$	**M1**	

QUESTION	SCHEME	MARKS	NOTES
	$(1-2x)^{-1} = 1 + 2x + 4x^2$	**M1**	
	One expansion correct and multiplied by their constant	**A1**	
	Adds the two series	**M1**	
	$1 + 8x + 10x^2$	**A1**	
(c)	$\lvert x \rvert < \frac{1}{2}$	**B1**	
	Total for Question 13	**(9)**	

QUESTION	SCHEME	MARKS	NOTES
14 (a)	$\frac{dx}{dt} = 2t,\ \frac{dy}{dt} = 3t^2 - 3$	**M1**	
	$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$	**M1**	
	$\frac{3t^2 - 3}{2t}$	**A1**	
(b)	Point $(4, 2)$ and gradient $\frac{9}{4}$	**M1**	
	$y - 2 = \frac{9}{4}(x - 4)$	**M1**	
	$9x - 4y - 28 = 0$	**A1**	
	Total for Question 14	**(6)**	

QUESTION	SCHEME	MARKS	NOTES
15 (a)	$t = \sqrt{3}$ and $t = -\sqrt{3}$ both give $(3, 0)$	**M1**	
	Two different values of t give the same point, so the curve crosses itself	**A1**	
(b)	$y = t(t^2 - 3)$	**M1**	
	$y^2 = t^2(t^2 - 3)^2$	**M1**	
	$= x(x - 3)^2$	**A1***	
(c)	Area $= 2\int_0^3 \sqrt{x}(3 - x)\,dx$ or $\left\lvert \int_{-\sqrt{3}}^{\sqrt{3}} y \frac{dx}{dt}\,dt \right\rvert$	**M1**	
	$3x^{\frac{1}{2}} - x^{\frac{3}{2}}$ or $2t^4 - 6t^2$	**M1**	
	Integrates: $2x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}}$ or $\frac{2}{5}t^5 - 2t^3$	**M1**	
	Correct integral	**A1**	
	Correct limits and doubling (or modulus)	**dM1**	
	$\frac{24\sqrt{3}}{5}$	**A1**	
	Total for Question 15	**(11)**	

TOTAL FOR PAPER: 100 MARKS