Name _____ Date _____ Score _____ / 100

LEMMA PRACTICE PAPER

Level 2

Pure Mathematics

Paper 2 · Standard

Paper reference **LP2/P** Time **2 hours**

TOTAL MARKS	100

You must have: pen, pencil, ruler, graph paper and a calculator.

INSTRUCTIONS

- Use black ink or ball-point pen; pencil may be used for diagrams.
- Answer **all** questions, in the spaces provided. If you need more space, continue on extra paper and label it clearly.
- Show all your working. An answer with no supporting method may get no marks.
- Give answers to 3 significant figures unless a question says otherwise, and give exact answers when asked.

INFORMATION

- There are **15** questions, and the paper is out of **100** marks.
- The marks for each part of a question are shown in brackets, for example **(3)**. Use them to judge how much to write.
- The questions are in roughly increasing order of difficulty.
- Difficulty: **Standard**. A typical exam paper. Familiar methods, a few multi-step questions and a handful of "show that"s. A solid student should score well with careful work.

ADVICE

- Read each question carefully before you start.
- Keep an eye on the time: roughly one minute per mark.
- Check your answers if you have time at the end.
- The mark scheme, very detailed model solutions and video explanations are free at **lemmamathematics.com/papers/pure-2**.

1 Find the set of values of x for which

(a) $x^2 - 2x - 15 > 0$, **(3)**

(b) $3x - 1 < 2x + 7$, **(1)**

(c) both $x^2 - 2x - 15 > 0$ and $3x - 1 < 2x + 7$, giving your answer in set notation. **(2)**

(Total for Question 1 is 6 marks)

2 Prove that, for all integers n, $n^3 - n$ is divisible by 6. **(4)**

(Total for Question 2 is 4 marks)

3 Show, by means of a counterexample, that the statement "$2^n + 1$ is prime for all positive integers n" is not true. **(2)**

(Total for Question 3 is 2 marks)

4 The functions f and g are defined by

$$f(x) = 2x + 3, \quad x \in \mathbb{R}, \qquad g(x) = x^2 - 1, \quad x \in \mathbb{R}.$$

(a) Find an expression for $fg(x)$. **(2)**

(b) Solve the equation $fg(x) = gf(x)$, giving your answers in simplest surd form. **(4)**

(c) Find $f^{-1}(x)$. **(2)**

(Total for Question 4 is 8 marks)

5 An arithmetic series has first term a and common difference d. The 5th term is 17 and the sum of the first 10 terms is 185.

(a) Find the values of a and d. **(4)**

(b) Find the least value of n for which the sum of the first n terms exceeds 1000. **(4)**

(Total for Question 5 is 8 marks)

6 A geometric series has first term 80 and common ratio r, where $0 < r < 1$. The sum to infinity of the series is 320.

(a) Show that $r = \frac{3}{4}$. **(2)**

(b) Find the 4th term of the series. **(2)**

(c) Find the smallest value of n for which the sum to infinity exceeds the sum of the first n terms by less than 1. **(4)**

(Total for Question 6 is 8 marks)

7 The value, £V, of a car t years after it was first sold is modelled by

$$V = 18\,000e^{-0.15t} + 2000, \qquad t \geq 0.$$

(a) State the value of the car when it was first sold. **(1)**

(b) Find the value of the car after 5 years, to the nearest pound. **(2)**

(c) Find the value of t when the car's value first falls below £6000. Give your answer to 2 decimal places. **(3)**

(d) Find the rate at which the value of the car is decreasing when $t = 5$. **(3)**

(e) State the long-term value of the car according to this model. **(1)**

(Total for Question 7 is 10 marks)

8 **(a)** Show that **(3)**

$$\frac{1-\cos 2\theta}{\sin 2\theta} \equiv \tan\theta.$$

(b) Solve, for $0 \leq \theta \leq \pi$, **(5)**

$$3\sin 2\theta = 2\cos\theta,$$

giving your answers to 3 decimal places where appropriate.

(Total for Question 8 is 8 marks)

9 Prove, from first principles, that the derivative of x^2 is $2x$. **(4)**

(Total for Question 9 is 4 marks)

10 The curve C has equation $y = x^3 - 4x$. The point $P(1, -3)$ lies on C. **(6)**

Find an equation of the tangent to C at P, and find the coordinates of the point where this tangent meets C again.

(Total for Question 10 is 6 marks)

11 **(a)** Use integration by parts to find $\int xe^{2x}\,dx$. **(4)**

(b) Hence find the exact value of $\int_0^1 xe^{2x}\,dx$. **(3)**

(Total for Question 11 is 7 marks)

12 Find $\int 6x(x^2+1)^2\,dx$. **(3)**

(Total for Question 12 is 3 marks)

13

$$f(x) = \frac{7x + 1}{(1 + x)(1 - 2x)}$$

(a) Express $f(x)$ in partial fractions. **(3)**

(b) Hence find the series expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2, simplifying each term. **(5)**

(c) State the range of values of x for which your expansion is valid. **(1)**

(Total for Question 13 is 9 marks)

14 The curve C has parametric equations

$$x = t^2, \qquad y = t^3 - 3t, \qquad t \in \mathbb{R}.$$

(a) Find $\dfrac{dy}{dx}$ in terms of t. **(3)**

(b) Find an equation of the tangent to C at the point where $t = 2$, in the form $ax + by + c = 0$. **(3)**

(Total for Question 14 is 6 marks)

15 The curve C has parametric equations

$$x = t^2, \qquad y = t^3 - 3t, \qquad t \in \mathbb{R}.$$

(a) Show that C crosses itself at the point $(3, 0)$. **(2)**

(b) Show that a Cartesian equation of C is $y^2 = x(x - 3)^2$. **(3)**

The curve C forms a closed loop between $x = 0$ and $x = 3$.

(c) Find the exact area enclosed by the loop. **(6)**

(Total for Question 15 is 11 marks)

TOTAL FOR PAPER IS 100 MARKS

END OF PAPER