LEMMA PRACTICE PAPER · MARK SCHEME

Level 3

Pure Mathematics

Paper 3 · Challenging exam

GENERAL MARKING GUIDANCE

- All candidates must receive the same treatment: mark positively, and give credit for what is shown rather than penalising what is missing.
- **M** marks: method marks, for knowing and applying a correct method to the problem, even if there are small slips.
- **A** marks: accuracy marks, which can only be awarded if the relevant M mark has been earned.
- **B** marks: unconditional accuracy marks, independent of method.
- **dM**: a method mark that depends on the previous M mark. **ft**: follow through from an earlier error. *****: the answer is given in the question, so full working must be seen with no errors. **cao**: correct answer only.
- Correct alternative methods earn full credit. The model solutions show several of them.

QUESTION	SCHEME	MARKS	NOTES
1	Uses $b^2 - 4ac < 0$	**M1**	
	$16 - 4k(k+3) < 0$	**A1**	
	$k^2 + 3k - 4 > 0$	**A1**	
	Critical values $k = -4,\ k = 1$ and chooses the outside region	**M1**	
	$k < -4$ or $k > 1$	**A1**	
	Total for Question 1	**(5)**	

QUESTION	SCHEME	MARKS	NOTES
2	Assumes $\log_2 3 = \frac{p}{q}$ with p, q positive integers	**B1**	Must say "assume, for contradiction".
	$2^{\frac{p}{q}} = 3 \Rightarrow 2^p = 3^q$	**M1**	
	Notes 2^p is even (as $p \geq 1$) and 3^q is odd	**M1**	
	Identifies the contradiction: an even number cannot equal an odd number	**A1**	
	Concludes that $\log_2 3$ is irrational	**A1**	
	Total for Question 2	**(5)**	

QUESTION	SCHEME	MARKS	NOTES
3 (a)	$na = 12$	**M1**	

QUESTION	SCHEME	MARKS	NOTES
	$\frac{n(n-1)}{2}a^2 = 60$	**M1**	
	Eliminates a, e.g. $\frac{n-1}{n} = \frac{120}{144}$	**M1**	
	$n = 6$	**A1**	
	$a = 2$	**A1**	
(b)	$\binom{6}{3}a^3$	**M1**	
	160	**A1**	
	Total for Question 3	**(7)**	

QUESTION	SCHEME	MARKS	NOTES
4 (a)	V shape with vertex $\left(\frac{3}{2}, 0\right)$ and y-intercept $(0, 3)$	**B1**	
	Line with intercepts $(0, 5)$ and $(5, 0)$	**B1**	
(b)	Solves $2x - 3 = 5 - x$ or $-(2x - 3) = 5 - x$	**M1**	
	$x = \frac{8}{3}$	**A1**	
	$x = -2$	**A1**	
(c)	Chooses the outside region from their critical values	**M1**	
	$x < -2$ or $x > \frac{8}{3}$	**A1**	
	Total for Question 4	**(7)**	

QUESTION	SCHEME	MARKS	NOTES
5 (a)	$R = 5$	**B1**	
	$\tan\alpha = \frac{4}{3}$	**M1**	
	$\alpha = 0.9273$	**A1**	
(b)	$\sin(\theta - \alpha) = \frac{1}{2}$	**M1**	
	$\theta - \alpha = \frac{\pi}{6}$ or $\frac{5\pi}{6}$	**M1**	
	$\theta = 1.451,\ 3.545$	**A1**	
	Total for Question 5	**(6)**	

QUESTION	SCHEME	MARKS	NOTES
6	Denominator minimum $7 - 5 = 2$	**M1**	
	Maximum value 5	**A1**	
	$\theta = \alpha + \frac{3\pi}{2} = 5.640$	**A1**	
	Total for Question 6	**(3)**	

QUESTION	SCHEME	MARKS	NOTES
7 (a)	$\ln y = \ln a + x \ln b$	**M1**	
	Compares with $Y = mX + c$: gradient $\ln b$, intercept $\ln a$	**A1**	
(b)	$\ln a = 1.2$	**M1**	
	$a = 3.32$	**A1**	
	$\ln b = \dfrac{3.7 - 1.2}{5} = 0.5$	**M1**	
	$b = 1.65$	**A1**	
(c)	$\ln y = 1.2 + 0.5 \times 8$	**M1**	
	$y = 181$	**A1**	
	Extrapolation, so it may be unreliable	**B1**	
	Total for Question 7	**(9)**	

QUESTION	SCHEME	MARKS	NOTES
8 (a)	$\dfrac{dV}{dr} = 4\pi r^2$	**B1**	
	$\dfrac{dr}{dt} = \dfrac{dV}{dt} \div \dfrac{dV}{dr}$	**M1**	
	$\dfrac{20}{4\pi \times 25}$	**M1**	
	0.0637 cm s^{-1}	**A1**	
(b)	$\dfrac{dS}{dr} = 8\pi r$	**B1**	
	$\dfrac{dS}{dt} = 8\pi r \times \dfrac{20}{4\pi r^2}$	**M1**	
	$= \dfrac{40}{r}$	**A1***	
(c)	$\dfrac{40}{r} = 2$	**M1**	
	$r = 20$ cm	**A1**	
	Total for Question 8	**(9)**	

QUESTION	SCHEME	MARKS	NOTES
9	$x = u^2,\ dx = 2u\,du$	**M1**	
	$\displaystyle\int \frac{2u}{u^2 + u}\,du$	**M1**	
	$\displaystyle\int \frac{2}{u + 1}\,du$	**A1**	
	$2\ln(u + 1)$	**M1**	
	Correct new limits $u = 1,\ 2$	**M1**	
	$2\ln 3 - 2\ln 2 = \ln\dfrac{9}{4}$	**A1***	

QUESTION	SCHEME	MARKS	NOTES
	Total for Question 9	**(6)**	

QUESTION	SCHEME	MARKS	NOTES
10	Parts with $u = x$: $-x\cos x + \int \cos x\, dx$	**M1**	
	$-x\cos x + \sin x$	**A1**	
	Substitutes limits	**M1**	
	1	**A1**	
	Total for Question 10	**(4)**	

QUESTION	SCHEME	MARKS	NOTES
11 (a)	$\overrightarrow{AB} = 2\mathbf{i} + 4\mathbf{j} + 4\mathbf{k}$	**M1**	
	6	**A1**	
(b)	$\overrightarrow{AC} = -2\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}$	**M1**	
	$\cos\theta = \dfrac{28}{6\sqrt{44}}$	**M1**	
	45.3°	**A1**	
(c)	$\frac{1}{2} \times 6 \times \sqrt{44}\sin\theta$ or $\frac{1}{2}\lvert\overrightarrow{AB} \times \overrightarrow{AC}\rvert$	**M1**	
	$10\sqrt{2}$	**A1**	
	Total for Question 11	**(7)**	

QUESTION	SCHEME	MARKS	NOTES
12 (a)	$\mathbf{d} = \mathbf{a} + \mathbf{c} - \mathbf{b}$	**M1**	
	$-2\mathbf{i} - \mathbf{j} + \mathbf{k}$	**A1**	
(b)	Height $= \dfrac{2 \times \text{area}}{\lvert AB\rvert}$	**M1**	
	$\dfrac{10\sqrt{2}}{3}$	**A1**	
	Total for Question 12	**(4)**	

QUESTION	SCHEME	MARKS	NOTES
13 (a)	Separates: $\int h^{-\frac{1}{2}}\, dh = \int -k\, dt$	**M1**	
	$2\sqrt{h} = -kt + c$	**A1**	
	$t = 0,\ h = 16 \Rightarrow c = 8$	**M1**	
	$t = 10,\ h = 9 \Rightarrow 6 = 8 - 10k$	**M1**	
	$k = 0.2$, giving $2\sqrt{h} = 8 - 0.2t$	**A1***	
(b)	$h = 0 \Rightarrow 8 - 0.2t = 0$	**M1**	

QUESTION	SCHEME	MARKS	NOTES
	40 minutes	**A1**	
(c)	$4 = 8 - 0.2t$	**M1**	
	20 minutes	**A1**	
(d)	Any sensible criticism	**B1**	E.g. the rate may not follow $\sqrt{h}$ exactly near the bottom; the shape of the tank isn't considered.
	Total for Question 13	**(10)**	

QUESTION	SCHEME	MARKS	NOTES
14 (a)	Product rule: $2xe^{-x} - (x^2 - 3)e^{-x}$	**M1**	
	$e^{-x}(-x^2 + 2x + 3)$	**A1**	
	$-(x - 3)(x + 1)e^{-x}$	**A1***	
(b)	$x = 3,\ x = -1$	**M1**	
	$(3, 6e^{-3})$ and $(-1, -2e)$	**A1**	
	Sign of $\dfrac{dy}{dx}$ either side, or second derivative	**M1**	
	$(-1, -2e)$ minimum, $(3, 6e^{-3})$ maximum	**A1**	
	Total for Question 14	**(7)**	

QUESTION	SCHEME	MARKS	NOTES
15 (a)	Roots $x = \pm\sqrt{3}$	**B1**	
	Parts on x^2e^{-x} (applied twice), or a valid antiderivative method	**M1**	
	$\int x^2e^{-x}\,dx = -e^{-x}(x^2 + 2x + 2)$	**A1**	
	Antiderivative $F(x) = -e^{-x}(x^2 + 2x - 1)$	**A1**	
	Substitutes $\pm\sqrt{3}$	**M1**	
	$F(\sqrt{3}) = -(2 + 2\sqrt{3})e^{-\sqrt{3}}$ and $F(-\sqrt{3}) = (2\sqrt{3} - 2)e^{\sqrt{3}}$	**A1**	
	Takes the modulus (region below the axis)	**M1**	
	$(2\sqrt{3} - 2)e^{\sqrt{3}} + (2\sqrt{3} + 2)e^{-\sqrt{3}}$	**A1**	≈ 9.24
(b)	Uses the behaviour as $x \to \pm\infty$	**M1**	$y \to \infty$ as $x \to -\infty$; $y \to 0^+$ as $x \to \infty$.
	Upper bound $6e^{-3}$	**A1**	
	$0 < k < 6e^{-3}$	**A1**	
	Total for Question 15	**(11)**	

TOTAL FOR PAPER: 100 MARKS